The copy filmed here has been reproduced thanks to the generosity of:

University of Toronto Archives

The images appearing here are the best quality possible considering the condition and legibility of the original copy and in keeping with the filming contract specifications.

Original copies in printed paper covers are filmed beginning with the front cover and ending on the last page with a printed or illustrated impression, or the back cover when appropriate. All other original copies are filmed beginning on the first page with a printed or illustrated impression, and ending on the last page with a printed or illustrated impression.

The last recorded frame on each microfiche shall contain the symbol → (meaning "CONTINUED"), or the symbol ▼ (meaning "END"), whichever applies.

Maps, plates, charts, etc., may be filmed at different reduction ratios. Those too large to be entirely included in one exposure are filmed beginning in the upper left hand corner, left to right and top to bottom, as many frames as required. The following diagrams illustrate the method:

L'exemplaire filmé fut reproduit grâce à la générosité de:

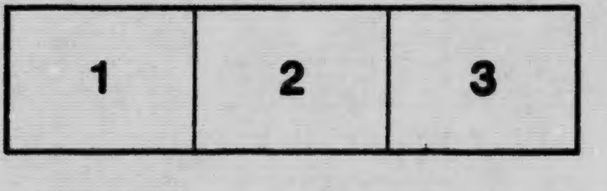
University of Toronto Archives

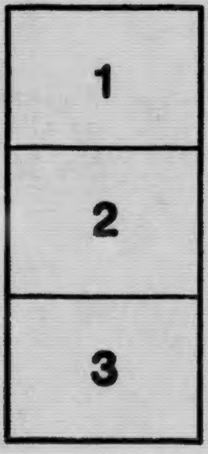
Les images suivantes ont été reproduites avec le plus grand soin, compte tenu de la condition et de la netteté de l'exemplaire filmé, et en conformité avec les conditions du contrat de filmage.

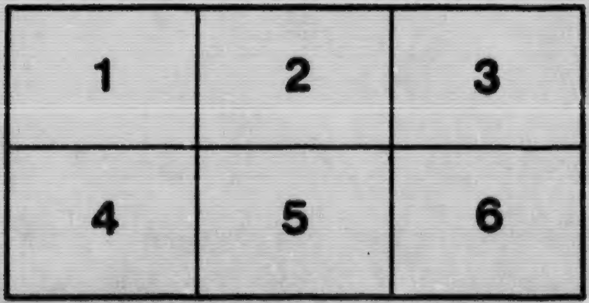
Les exemplaires originaux dont la couverture en papier est imprimée sont filmés en commençant par le premier plat et en terminant soit par la dernière page qui comporte une empreinte d'impression ou d'illustration, soit par le second plat, selon le cas. Tous les autres exemplaires originaux sont filmés en commençant par la première page qui comporte une empreinte d'impression ou d'illustration et en terminant par la dernière page qui comporte une telle empreinte.

Un des symboles suivants apparaîtra sur la dernière image de chaque microfiche, selon le cas: le symbole → signifie "A SUIVRE", le symbole ▼ signifie "FIN".

Les cartes, planches, tableaux, etc., peuvent être filmés à des taux de réduction différents. Lorsque le document est trop grand pour être reproduit en un seul cliché, il est filmé à partir de l'angle supérieur gauche, de gauche à droite, et de haut en bas, en prenant le nombre d'images nécessaire. Les diagrammes suivants illustrent la méthode.







UNIVERSITY OF TORONTO STUDIES

PAPERS FROM THE PHYSICAL LABORATORIES

No. 58: IMAGE FORMATION BY CRYSTALLINE MEDIA,

BY H. F. DAWES

(REPRINTED FROM THE PHILOSOPHICAL MAGAZINE, VOL. XXXII)

APR [illegible] 1957
UNIVERSITY

THE UNIVERSITY LIBRARY: PUBLISHED BY
THE LIBRARIAN, 1918

University of Toronto Studies
COMMITTEE OF MANAGEMENT

Chairman: Sir Robert Alexander Falconer, LL.D., K.C.M.G.
President of the University

Professor W. J. Alexander, Ph.D.
Professor W. H. Ellis, M.A., M.D.
Professor J. J. Mackenzie, B.A., M.B.
Professor J. P. McMurrich, Ph.D.
Professor G. H. Needler, Ph.D.
Professor George M. Wrong, M.A.

General Editor: H. H. Langton, M.A.
Librarian of the University

From the PHILOSOPHICAL MAGAZINE, vol. xxxii. *August* 1916.

Image Formation by Crystalline Media. By H. F. DAWES, *Professor of Physics, McMaster University, Toronto, Canada* *.

IN 1877 Stokes † discussed the question of the position of the image (the "apparent depth" of an object-point) due to the refraction of light from a crystal to air through a cleavage-plane or other plane refracting surface. This paper was written with the object of developing a method of testing crystals by applying the then recently discovered microscope method of determining the index of refraction, and was tested out for a great many specimens by Sorby ‡.

Stokes's investigation does not seem to have been very generally followed up or applied from an optical point of view, although the results are particularly interesting in the study of the optical properties of crystals. In fact, the only record of the application of the principles developed which I have been able to find is in Clay's admirable collection of Experiments in Light. Independent of these sources, however, the writer developed in 1908 a simple laboratory experiment of the type under consideration for use in his laboratory classes at the Physical Laboratory of the University of Toronto. In addition to the phenomena considered by Stokes there are further cases of image formation which are interesting and important, and the writer proposes to consider certain of these.

* Communicated by Professor J. C. McLennan, F.R.S.

† G. G. Stokes, Proc. Roy. Soc. xxvi. p. 386 (1877).

‡ Sorby, *ibid.* p. 384.

General Considerations.

The ordinary light in uniaxial crystals is propagated in precisely the same way as in ordinary isotropic substances, so that the laws for image-formation by this light are well known and need no further discussion. It is the extraordinary images in these crystals and both images in biaxials which follow more complex laws,—due to the more complicated shape of the wave-front and the fact that the rays are not necessarily at right angles to the waves. In investigating the laws for the image-formation in such cases, therefore, it is necessary to take into account the curvatures of the incident wave as well as the nature of the refracting surface and the change of velocity on refraction. The image is in general astigmatic, so that the investigation involves the determination of the positions and directions of two focal lines.

As an example of the method which must be employed, consider the formation of the image of a point-source within a uniaxial crystal by the extraordinary light refracted into air through a plane surface which is normal to the optic axis of the crystal. Such an image is intended to be seen by the eye or a low-power microscope, so that the pencil of rays coming from object or image-point is of small angular aperture. For this reason the standard approximations of the "first order" discussion of image-formation are justified and will be assumed without comment. The procedure here followed is not the same as that of Stokes, although it amounts to the same thing in the end.

As is well known *, this extraordinary wave in a uniaxial crystal travels out from a point-source in the form of a spheroid whose axis of revolution is the optic axis of the crystal. The following are geometrical and optical properties of such a wave-front.

The principal indices of refraction are n_1 equal to V/a, and n_2 equal to V/c, (1), where V is the velocity of light in air (or vacuum), a is the velocity of the extraordinary wave along the axis, and c its velocity at right angles to the axis; a is also the velocity of the ordinary wave.

If a wave has travelled a distance x along the axis it has also travelled a distance y at right angles to the axis such that $y=cx/a$, (2), so that the semi-axes of the wave-surface are x, y, y. The radius of curvature of the spheroid in any normal section at its pole is

---

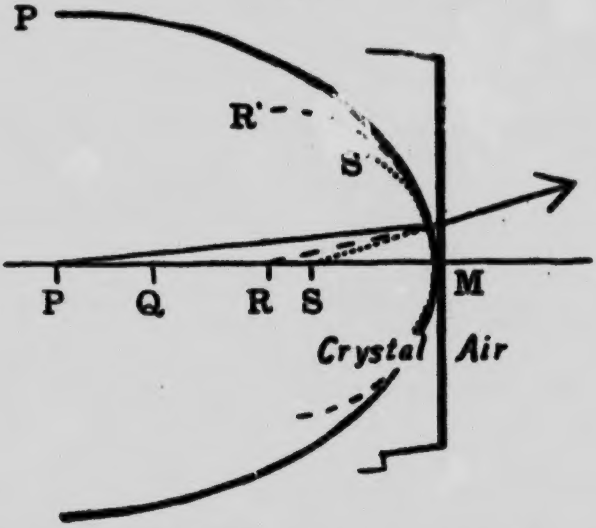
* Drude, 'Theory of Optics,' Properties of Transparent Crystals, from (55) setting b equal to c. Further relations are quoted or are deduced very directly from results in the same chapter; they will be referred to by the number of the equation which represents them.

$$r=y^2/$$ [illegible] $$c^2x/$$ [illegible] $$=n_1^2x/n_2^2, \quad (3).$$

The diagrams are drawn to correspond to a crystal of principal indices $n_1=4/3$ and $n_2=2$. The trace of the wave is shown at incidence and just after refraction; also the circle of curvature of the elliptic wave is shown in certain cases. The traces for actual wave-positions are indicated by continuous lines and for "virtual" wave-positions by dotted lines.

For the example under consideration the source of light P lies within the crystal at a distance p from the refracting surface at M (fig. 1). The ordinary waves form an image

Fig. 1.



at a distance q from M such that p/q is equal to n_1, (4), as in ordinary isotropic substances. The extraordinary waves also form an image, but it is not situated at Q, although the two sets of waves travel with the same velocity along PM. (This is contrary to the statement usually made in discussions of this subject, viz., that only one image is seen on looking along the axis into a crystal. It is true that the eye seems to see only one image, but this is because the one image is vertically beneath the other and the depth of focus of the eye is very great compared with the distance between the images. The presence of the two images may, however, be readily demonstrated by means of a microscope; for example, in the case of calcspar it is necessary to move the microscope along its axis a distance of about one-seventh of the depth of the object-point in the crystal in order to change the focus from the one image to the other.)

The formation of this second image arises from the fact that although the waves have equal velocities along the axis they have not the same curvatures, and the position of the image is determined not only by the change of velocity on refraction but also by the change of curvature. Since the wave has travelled a distance p along the axis, the radius of curvature of the wave-front is c^2p/a^2 as it reaches the refracting surface, by (3). Now, when refraction takes place at a plane refracting surface, the ratio of the curvatures of the incident and refracted waves is equal to the ratio of the velocities of light in the two media. Hence the radius of curvature of the wave as it enters the air is given by

$$\frac{s}{c^2p/a^2}=\frac{a}{\mathrm{V}}, \text{ i. e. } s=\frac{c^2p}{a\mathrm{V}};$$

and since the image in air is situated at the centre of curvature S of the waves coming from it, the distance s must be the required distance from the refracting surface to the image. In the diagram, P′M is the incident wave, R′M its circle of curvature, and S′M the refracted wave.

These results may be used to measure the principal extraordinary index of the crystal by the microscope method. Thus, s/p which is equal to $c^2/a\mathrm{V}$ is also equal to n_1/n_2^2, (5), by (1). The image is therefore at the same depth as it would be in a substance of index n_2^2/n_1, and the value of n_2 is given by

$$n_2=\frac{n_1}{s/p} \text{ or } \frac{p^2}{qs}, \text{ by (4).}$$

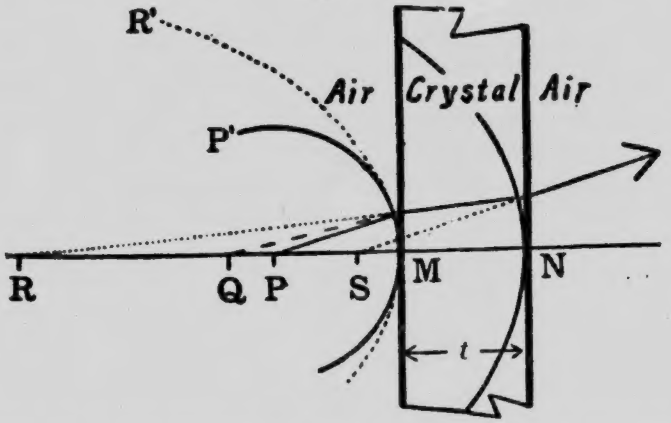
The values of p, q, and s will then be found by focussing the microscope in turn on P, Q, and S, and finally on M.

No one appears to have called attention to the fact that the two images cannot be distinguished from one another by the usual polarization test if the eye or microscope is placed symmetrically over the axis of the crystal through the source. For, in the wave-front of the extraordinary pencil the vibration directions radiate out equally in all directions from the axis, so that an analyser will transmit the same amount of light in all azimuths and the source will appear to be of the same brightness for all positions of the analyser. On the other hand, in the wave-front of the ordinary pencil the vibrations are uniformly transversal about the axis as centre, and therefore the image formed by this pencil will also seem of constant brightness as the analyser is rotated. If, however, there is any departure from the symmetry of the position of the eye, the brightness of the images will change as the analyser is rotated, so that the images may be distinguished. Even in this case there is no position of the analyser which will produce complete extinction of either image.

IMAGE FORMATION UNDER OTHER TYPES OF CIRCUMSTANCES.

1. *Image of a point in air by light refracted into a uniaxial crystal through a plane interface normal to the optic axis.*— The position of the image in this case may be determined by a process similar to the above. Thus, in fig. 2, with the source at P, the curvature of the wave P′M before refraction is $1/p$; after refraction it will be a/Vp. But after refraction the wave will proceed as part of a spheroid R′M whose principal curvature has this value, the rays radiating from the centre R of the spheroid. Since the required image is at the point from which the light thus radiates, its position is this centre and its distance from the refracting surface is the corresponding semi-axis of the spheroid. In accordance with (3) this distance is

Fig. 2.



$$r=\frac{a^2 Vp}{c^2 a},\ (6),$$

or, in terms of the indices, $n_2^{\,2}p/n_1$. This distance also is the same as if the light were being refracted into a substance of index n_2^2/n_1.

2. *Image formed by direct refraction through a plane parallel plate of uniaxial crystal of thickness* t, *the faces being normal to the axis.*—The light is refracted into the crystal in accordance with the law developed in the preceding section, and is refracted out according to that of the first example considered. Thus, if the source of light is at P in fig. 2 at the distance p from the first surface M, the virtual source of the refracted pencil is R at the distance n_2^2p/n_1 from that surface (as in Section 1) and consequently at the distance $t+n_2^2p/n_1$ from N at the second surface of the plate. The image formed by the second refraction is therefore by (5) situated at S at the distance $p+tn_1/n_2^2$ from N.

The distance from the final image to the original source is $t(1-n_1/n_2^2)$, (7), so that the image is "drawn up" toward the observer by this amount, which is the same as for an isotropic plate of index n_2^2/n_1 and is independent of the distance of the source.

3. *Image formed by direct refraction from one uniaxial crystal to another through a plane surface of separation which is normal to the axis of each crystal.*—The set of waves forming the image is extraordinary in both crystals, since the plane of incidence and refraction is a principal section in each.

One may deduce the law for the image position in this case from the laws developed for refraction from air to crystal and from crystal to air. Suppose for the moment that the two crystals are separated by a plane parallel air-space of thickness t, and that the light is refracted from crystal 1 into this air-space and then into crystal 2. According to (5) the image formed by the first refraction is situated at the distance $n_1'p/n_2'^2$ from the first refracting surface, and therefore $(t+n_1'p/n_2'^2)$ from the second. Since this image forms the source with reference to the second surface, it follows by (6) that the distance of the final image from this surface is $(t+n_1'p/n_2'^2)n_2''^2/n_1''$.

Now the *direction* of the final part of any of the rays forming this image is independent of the thickness of the air-space, and is therefore the same as if the thickness were indefinitely diminished, *i. e.*, as if the crystals were in optical contact. Hence the required image position, which is determined by the direction of the rays for optical contact, is the limiting position of the above image when the value of t is reduced to zero. Its distance from the surface of contact will therefore be

$$p\frac{n_2''^2}{n_1''}\Big/\frac{n_2'^2}{n_1'}.$$

As in other cases considered, this distance is just the same as if the light were refracted from a substance of absolute refractive index $n_2'^2/n_1'$ to one of index $n_2''^2/n_1''$.

4. *Image formed by direct refraction through a series of plane parallel plates of uniaxial crystal all having optic axes in the common normal to the surfaces.*—The extraordinary wave in the first crystal will be extraordinary in all successive plates and the ordinary will remain ordinary, so that only two images will be formed at each stage.

By the argument of Section 3 the image-position at each stage and therefore the final image-position will be the same as if the light were refracted into and out of a thin air-film of negligible thickness separating each pair of plates. By (7) the image is "drawn up" toward the observer a distance $t(1-n_1/n_2^2)$, on account of refraction through any plate of thickness t and indices n_1 and n_2, and this "drawing up" process will be repeated for each plate of the series. Hence the image will be nearer to the observer than the original source by the sum of all such quantities as $t(1-n_1/n_2^2)$. The case of some plates being isotropic is of course included in this, the corresponding displacement terms being obtained by setting n_1 equal to n_2.

5. *Image-formation by "direct refraction" through a lens of uniaxial crystal with optic axis lying along the geometrical axis.*—Recall that for refraction from a medium of index n' to one of index n'' through a surface of curvature $1/r$, the object and image distances are related by the law

$$\frac{n''}{v}-\frac{n'}{u}=k, \quad (8),$$

where k is the dioptric power of the surface and is equal to $(n''-n')/r$, (9). Note that this is fundamentally a relation between the curvature $1/u$ of the incident wave and the curvature $1/v$ of the refracted wave.

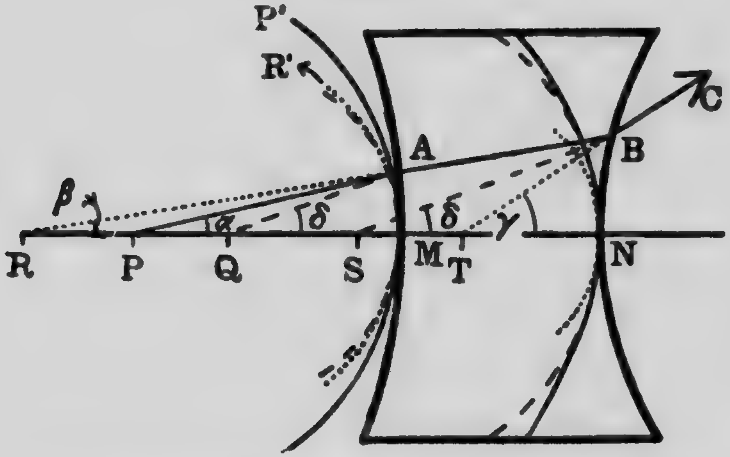
In the case under consideration, the light-wave P′M from a point P on the axis (fig. 3) is incident on the first surface M; is refracted as a spheroidal wave R′M whose centre of curvature is a certain point Q and centre of radiation the centre of the spheroid R; is incident on the second surface N as a wave with centre of curvature at a point S: is finally refracted into air as a homocentric pencil of centre T. T is the required image-point.

Since the light travels along the axis of the crystal with the velocity corresponding to the index n_1, the powers of the first and second surfaces will be k' and k'', equal to $(n_1-1)/r'$ and $(1-n_1)/r''$, by (9). In accordance with (3)

$$\mathrm{RM}=n_2^2\,.\,\mathrm{QM}/n_1^2 \text{ and } \mathrm{RN}=n_2^2\,.\,\mathrm{SN}/n_1^2, \quad (10).$$

Draw the ray PABC whose successive directions make the angles α, β, γ with the axis; the line QA makes the angle δ with the axis. These angles, much exaggerated in the diagram, are sufficiently small to admit of the ordinary approximations of the standard "first order" discussions of the principles of image-formation.

Fig. 3.



The line SB is parallel to QA. For the time required to travel from any one wave-position to any other is the same for all rays; and for any given ray this time is proportional to the distance between the waves. Hence, since A lies on the wave-front R'M from R and B lies on the corresponding wave-front through N, it follows that RA : RB :: RM : RN. But QA : SB in this same ratio by (10), so that the triangles RQA and RSB are similar.

The distances from A and B to the axis are

$$y' = \mathrm{PM} \,.\, \alpha = \mathrm{QM} \,.\, \delta = \mathrm{RM} \,.\, \beta,$$

and $$y'' = \mathrm{RN} \,.\, \beta = \mathrm{SN} \,.\, \delta = \mathrm{TN} \,.\, \gamma, \quad (11);$$

and, in view of these values, standard relations for the first and second refractions may be written from (8) as follows:

$$n_1\delta - \alpha = k'y', \quad (12),$$

and $$\gamma - n_1\delta = k''y'', \quad (13).$$

The problem now resolves itself into the elimination of the quantities which are involved in the use of particular rays and of the intermediate image formed within the medium of the lens. By this process the law for the image-position is determined in terms of the optical constants of the system and the position of the object. The method corresponds to the standard one of dealing with a thick lens. Thus:

$$\gamma-\alpha=k'y'+k''y''\text{, by addition, from (12, 13),}$$
$$=k'y'+k''y'+k''t\beta\text{, by (11),}$$
$$=k'y'+k''y'+k''tn_1^2\delta/n_2^2$$
$$=k'y'+k''y'+k''tn_1k'y'/n_2^2$$
$$+k''tn_1\alpha/n_2^2\text{, by (12).}$$

Hence

$$y/a=\mathrm{K}\ .\ \mathrm{PM}+(1+k''tn_1/n_2^2)\text{, (14),}$$

where K is the power of the system and is equal to

$$k'+k''+k'k''tn_1/n_2^2\text{, (15).}$$

Similarly,

$$\alpha/\gamma=-\mathrm{K}\ .\ \mathrm{TN}+(1+k'tn_1/n_2^2)\text{, (16).}$$

(14) and (16) state the fundamental general law for the image-position by expressing the distances from the refracting surfaces to object and image in terms of the ratio γ/α, which may be considered an arbitrary parameter. These formulæ are similar to the standard formulæ for a lens of isotropic substance *, the modification consisting essentially in that the distance travelled within the crystal is divided by n_2^2/n_1, corresponding to division by the index in ordinary lenses. The positions of the cardinal points of the lens and the lens formulæ of standard type may therefore be written down on this basis.

In addition to the image system formed by the light which traverses the lens as the extraordinary wave-front, there is an image system produced by the ordinary wave-front, and although the light traverses the lens in the neighbourhood of the optic axis the two image-systems are not coincident. The fundamental formulæ corresponding to (14, 16, 15) are for this system,

$$\gamma/\alpha=\mathrm{K}'\ .\ \mathrm{PM}+(1+k''t/n_1),\quad\text{(17),}$$
$$\alpha/\gamma=-\mathrm{K}'\ .\ \mathrm{TN}+(1+k't/n_1)\text{, (18),}$$

where K′ the power of the lens is

$$k'+k''+k'k''t/n_1\text{, (19).}$$

It will be seen that the difference in the two sets of formulæ lies in the terms which involve the thickness of the lens, so that the two images of an object coincide when and

* See Hermann, 'Geometrical Optics,' p. 57.

only when the thickness is negligible. In consequence of this it will not be possible with thick lenses of crystal to focus sharply either on a photographic plate or in the focal plane of an eyepiece. The two most important cases in which this may affect the efficiency of the lens are in its use as objective of telescope or collimator. In these cases it is the separation of the one set of principal foci which is important. This separation may be estimated as follows:—

The distances from the lens to the second principal foci are determined by setting the arbitrary parameter equal to zero in (16) and (18), giving

$$0=-K\ .\ FN+(1+k'tn_1/n_2^2),$$

$$0=-K'\ .\ F'N+(1+k't/n),$$

from which the distance between the foci is equal to

$$k'^2t(n_1^2-n_2^2)/n_2^2n_1KK'.$$

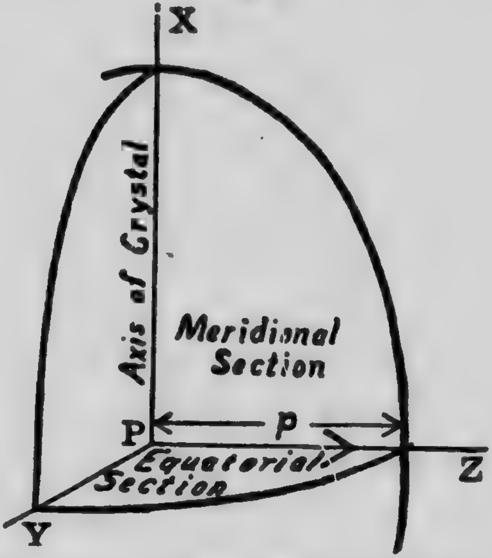
In a quartz lens of say about 20 cm. focal length and surfaces of equal curvature, this amounts to about ·001 of the thickness, and in a calcite lens of the same power to about ·037 of the thickness. The separation is thus unimportant in quartz lenses of less than about ·5 cm. thickness except for the highest magnifications.

This separation of the second principal foci disappears when the first surface of the lens is plane, so that plane waves are unchanged in shape until the second surface is passed, but in this case the separation of the first principal foci is proportionately greater on account of the fact that the whole of the dioptric power is produced at the one surface. Similar considerations would show that the separation of the first principal foci would be zero for a lens with the second surface plane. The fact of double refraction requires, therefore, that the objectives of collimator and telescope be plano-convex and that the lenses be placed with plane faces out.

6. *Image-formation by direct refraction by uniaxial crystals cut so that the optic axis is at right angles to the geometrical axis of the refracting system.*—For such examples of refraction of the extraordinary wave, the part of the spheroid presented to the refracting surface lies in the neighbourhood of zero latitude. The curvatures in normal sections in this part of the wave-front are unequal, so that on refraction by a symmetrical system an astigmatic image will be formed. Since the principal curvatures lie in and at right angles to

the plane of the meridian, the focal lines, which are determined by these curvatures, will be at right angles to and parallel to the axis of revolution, and hence at right angles to and parallel to the optic axis of the crystal. Thus in fig. 4, which represents the extraordinary wave from a point P, the optic axis of the crystal is parallel to the x-axis and the geometrical axis of the refracting system lies along the z-axis. The wave travels with the velocity a along the optic axis and with the velocity c at right angles to it, so that the section by the zy-plane is the equatorial circle and that by the zx-plane the meridional ellipse. The focal line corresponding to the former section is parallel to the x-axis and intersects the z-axis at the image-point, as determined for a spherical wave of the same curvature travelling with the same velocity in an isotropic substance. Similarly, the focal line corresponding to the meridional section is parallel to the y-axis, and its point of intersection with the z-axis is the same as if this wave were spheroidal about this line as axis of revolution. The position of this point of intersection will therefore be determined in accordance with the principles which we have been considering in previous sections; the necessary modification of the formulæ will consist simply in interchanging the indices n_1 and n_2, since the velocities a and c are now interchanged with reference to the incident wave-front.

Fig. 4.



Keeping these observations in mind, the following conclusions may be easily deduced :—

(1)* *For an object-point within a crystal at a distance* p *from a plane refracting surface.*

Distance from surface to :

(*a*) focal line parallel to axis . . . p/n_2;
(*b*) focal line at right angles to axis, pn_2/n_1^2;
(*c*) ordinary image . . . . p/n_1.

(2) *For a lens of crystal.*

The powers of the first and second surfaces and of the lens as a whole are :—

(*a*) for pencils of ordinary light :

$$k_1=\frac{n_1-1}{r},\quad k_2=\frac{1-n_1}{s},\quad K=k_1+k_2+k_1k_2t/n_1\,;$$

(*b*) for focal lines parallel to the optic axis of the crystal, as governed by the circular section of the wave-surface :

$$k_1'=\frac{n_2-1}{r},\quad k_2'=\frac{1-n_2}{s},\quad K'=k_1'+k_2'+k_1'k_2't/n_2\,;$$

(*c*) for focal lines at right angles to optic axis, governed by elliptic section of wave-front :

$$k_1''=\frac{n_2-1}{r},\quad k_2''=\frac{1-n_2}{s},\quad K''=k_1''+k_2''+k_1''k_2''tn_2/n_1^2.$$

Knowing the values of the powers for these cases, the positions of the sets of cardinal points of the lens and the formulæ relating object to image positions may be written down in accordance with the standard formulæ for these quantities,—it was shown in the discussion of Section 5 that these formulæ in a modified form apply to the refraction of the elliptic wave.

It will be seen that the powers of the individual refracting surfaces are the same for both the principal sections of the wave-front. The powers of the lens as a whole differ, however, in the term which involves the thickness, so that the amount of the astigmatism is governed by the thickness; if the thickness is negligible the focal lines degenerate into a point and the emerging pencil remains homocentric. The powers for the ordinary and extraordinary waves are, however, essentially different for each surface and for the lens as a whole. There are therefore always two image series for a lens of crystal cut as described in this section.

---

* Included in Stokes's discussion.

Crystalline Components in a Lens System.

Whenever a crystalline component,—lens or plane-parallel plate,—which produces two image series is incorporated into a lens system, the system as a whole will of course produce two series of images. For example, for a doublet consisting of a lens of crystal of powers K′ and K″ and a lens of glass of power K, the combination will have the powers K′+K and K″+K. It is evident that the relative values of the powers of such a combination may be made almost anything one wishes; one of the values may even be made zero by choosing the glass lens of power equal and opposite to that of the selected crystal power. In this case the other power will be the difference K′−K″ of the powers of the crystal lens.

Again, it will not be possible to produce an achromatic doublet which will at the same time correct the chromatic aberration of both powers of a lens of crystal. If ω', ω'', ω, are the dispersions for the ordinary and extraordinary powers of the crystalline lens and of the glass lens combined with it, and K′, K″, K the corresponding mean powers, the chromatic aberrations of the powers of the combination are

$$\omega' K' + \omega K, \text{ and } \omega'' K'' + \omega K.$$

If the values of ω and K are so chosen that one of these is zero in accordance with the ordinary principles of Geometrical Optics, the value of the other will be

$$\omega'' K'' - \omega' K',$$

and this will not likely happen to be zero. In the case of a lens of Iceland spar, of which the principal indices are about 1·66 and 1·49 and the dispersions for the D–F lines ·014 and ·0088, this outstanding chromatic aberration will be $-{\cdot}0051\ (1/r - 1/s)$, or about ·02 of the power of the combination if the glass lens has the index 1·62 and the dispersion ·02.

Summary.

Laws for the image position are developed for a number of typical cases of image formation by the extraordinary rays in uniaxial crystals.

It will be noticed that in the examples of light travelling along the axis of the crystal, the thickness of the crystal traversed enters into the expressions for image position as if the index were n_2^2/n_1 as compared with isotropic substances; n_1 and n_2 are the principal indices of the crystal. This quantity may be considered a sort of pseudo-index applicable to such cases.

It is shown that lenses of crystal will in general form two series of images corresponding to the two sets of waves; the laws for the image series due to the extraordinary waves correspond to those for lenses of isotropic substance, and the expressions are found for the dioptric power. The question of the use of crystal components in lens systems is also considered briefly.

Physical Laboratory,
University of Toronto,
May 2, 1916.